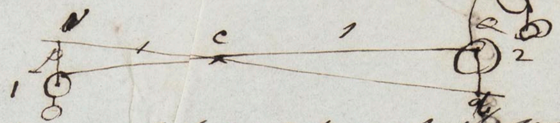
$x + x = 3p - 2x$
$p = 3x \quad x = \frac{p}{3}$

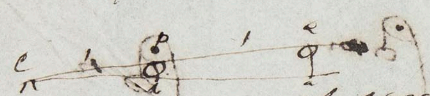
The Force of a particle of Matter in any proposed circumstances is

If a body or system of bodies be kept at rest by any obstacle opposed to the motion as by the hand by ~~opposite~~ weights &c acting over pulleys upon the removal of this obstacle motion takes place, & I define the force of each body to be ~~the~~ its ~~magnitude~~ quantity of matter into its actual velocity. & the direction of the force to be the instant direction of the body



For example let ~~Q~~ Q be double of P & at the same distance from C the cr of motion upon leaving the rod free, Q descends thro' $1/3$ a & P thro' an equal arc P^1 in the same time ∴ the force of P is $\frac{1}{2}$ that of Q & in a contrary direction

Again let Q be at double the distance on the same side of the Cr of motion. Q descends thro' [illegible] & P [illegible] in equal times ∴ the force of P is $\frac{1}{2}$ that of Q

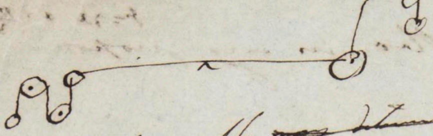


It is evident that this actual force of Bodies connected together in any [illegible] & [illegible] the force from [illegible] with they [illegible] circumstances [illegible] the [illegible] was dissolved.

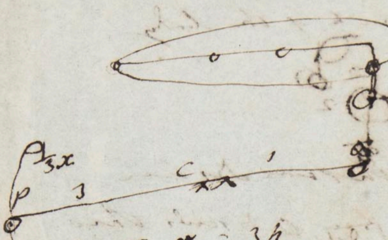
It may be thought to upset the action [illegible] as [illegible] the force. It may be said that in the ~~last~~ 1st case the force on P is really that of gravity [illegible] in the same direction the force on Q [illegible] 2 that the [illegible] acting on the [illegible] body P & Q is equal in both. but does not P [illegible] that case [illegible] contrary direct from that of Q. [illegible] of the connexion between P & Q tis true that the force of gravity would make them both descend, but in the actual circumstances there is something which makes P ascend. why should not I call that the exertion of a force [illegible] in the direction P b. there is something more than [illegible] which determines the motion of P. ~~[illegible]~~ whether [illegible] the causes exist, & how they be imagined & expressed, the result of all is motion in direction P b & that result I call the actual force on P. so in the 2d case P really moves ~~[illegible]~~ $\frac{1}{2}$ the instant velocity of Q & I call its force $\frac{1}{2}$ ~~[illegible]~~ of Q.

~~In that case~~ P & Q move in the same manner as they would setting aside gravity & the connexion of the bodies P was acted upon by a force P b & Q by a force Q a.

Consider then P & Q moving freely [illegible] the forces P a Q b equal forces in opposite directions [illegible] = Q would produce equilibrium & as the facts are the same [illegible] suppose P & Q thus free



which really ~~may be found by (~~ may move C by the power of 87 medium = friction
~~equal force~~ the force of P as ... other in this other case (or more properly speaking under this other idea of the cause of P, & Q, motion)



$3x\,p.\,x$ $\quad r = \frac{3p}{5}$

$3xp = r$ $\quad x = \frac{1}{5}$

$xp = \frac{r}{3}$

$\overline{p - r} = p + \frac{r}{3}$

$\frac{2r}{3} = \quad 3p - p = \frac{10r}{3}$

$p = \frac{5r}{3}$ Divide moving force by inertia

$r = \frac{3p}{5}$

$\frac{\frac{2p}{3} \times 3}{p \times 3 + p}$

$p + 10$

Let us take then the case of P, Q acted on by gravity & the acc. by gravity (g) I'll call
~~moving carrying the~~ on the same side of the C. Let the
actg force of P be x then that of Q is $x \times \frac{CQ}{CP}$.
the moving force of P is $P \times x$ of Q is $\frac{Qx \cdot CQ}{CP}$

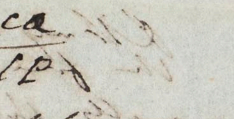
Let a wt ~~m~~ whose moving force is mG & is equal to $P \times PA$
of P be suspended from by means of a pully or otherwise & act directly
against P. in the same manner let a wt n whose moving force
nG is $= Qx\frac{CQ}{CP}$ act agst Q. then by the opposition of
equal forces the wt be = brium. but by the doctrine of = brium
we know that $PC \cdot P \times \cancel{x} - \cancel{PCm} \cancel{x} + \cancel{QCQ} - CQnG = 0$ which equation
gives x in terms of G &c, the mass of P, Q & their distances from C

$Gm = Px = Gn = Qx \cdot \frac{CQ}{CP} = \frac{Gm Q}{P} \cdot \frac{CQ}{CP}$

$PC \cdot P + QQC = \frac{PC + QC}{m + n} = m \times \overline{PC - \frac{Q}{P} \frac{CQ^2}{CP}}$

$m = \frac{PA\,QCQ}{PC + \frac{Q}{P} \cdot \frac{CQ^2}{CP}}$

$x = \frac{Gm}{P} = \frac{\overset{PC}{P} + \overset{QC}{Q} \cdot G}{P \cdot PC + Q \cdot \frac{CQ^2}{CP}} \times G$

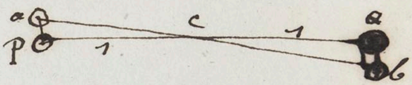


$an - bv$

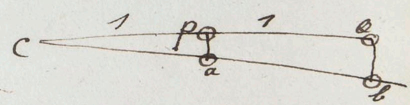
$\frac{Px}{G}$

1 If a body or system of bodies be kept at rest by any obstacle opposed to their motions, as by the hand for example, or by weights acting over pullies, or any other way whatsoever, upon the removal of this obstacle motion ensues, & I define ~~that~~ the force of each body to be its quantity of matter * into its initial velocity; & the initial direction of the force to be the initial direction of the body.

For example let Q be double of P & at ~~the~~ an equal distance of from the center (C) of motion. Upon leaving the horizontal rod to free ~~descend~~ move ~~freely~~ by the operation of gravity, Q descends thro Qb, and P ascends thro an equal space pa in the same time; ∴ by the defn the force of on P is ~~equal~~ ½ ~~to~~ that of on Q & in a contrary direction.



Again let P & Q be equal & let Q be at double the distance of P & on the same side of the center of motion. Q descends thro Qb, & P in the same time thro Pa which is ½ ~~that~~ Qb therefore the force on P is ½ that on Q.

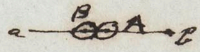


* or ~~force~~ ∝ Mass × ~~limit of~~ space ~~if~~ ~~x~~ ~~=~~ space moved thro in time t, & t be supposed diminished ad infinitum. Force ~~∝~~ varies as M × b/t [illegible] for the sum of the [illegible] of ~~smaller~~ [illegible] for the [illegible] being taken the same in comparing different forces.

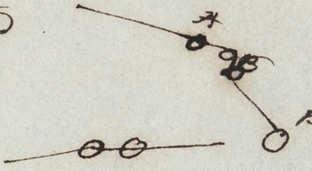
2

That is if bodies whose moving forces are = oppose each other's motion, there is an equilibrium

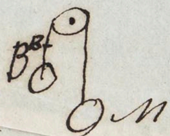
& for example by supposing A & B contiguous & the cohesion of a plane between them or by supposing A & B to be placed so placed that by their impenetrability each if actually to move, would carry the other with it.



or by supposing the ends connected by a flexible thread (fig 4 & 5) whose placed over perfect pullies so that the its extremities at A & B move in the direction of A & B motion



Ex. Let the bodies B & M [illegible] by means of a flexible string, & suppose the initial velocity with which B ascends rep'd by B b [illegible]



& let A & N be attach'd to the inflexible rod & moreover suppose A & N & the CN to revolve about C. & moreover suppose A & N & their initial distances from C to be so adjusted that A × their initial vel'y = B × Bb in vel — or if the rod describes thro' the initial [illegible] NCR while B ascends the inc. space Bb. that A Aa = B × Bb. Then we have A & B put into circumstances exactly equal.

In this manner it works
Otherwise thus

(Note) if we resolve this tendency to believe truths universally whether fixed truths & particular, into a love of order, it will be another instance of our love of generalization.)

The masters of who have ~~taught~~ delivered the elements of the sciences have had reasons for their choice of hypotheses ~~that~~ which are not always ~~apparent at~~ easily discerned ~~by it~~ in its enumeration. ~~& which it~~ it were to be wished they had sometimes delivered at large. ~~from this cause~~ for want of this it is owing that excellent axioms are sometimes rejected & incomparably inferior ones substituted in their place, & that even by the ~~skilful~~ instructed.

Among the various axioms that have been imagined for explaining the ~~property~~ equilibrium of the lever, none seems to me more ~~remarkable~~ either for real excellence, or for its apparent remoteness from common & popular ideas on the subject, than the following used by Newton.

It asserts this that if ~~a force~~ in the lever when there is a balance, ~~the force anywhere~~ there would still be a balance if the force (on either side) were removed ~~from~~ to any other point in the line of direction & there acted ~~as ...~~ on the branch of the lever proceeding from the centre to this new point. i.e. ~~substitute~~ suppose the force & the arm it acts on, & substitute the same force in any other point of the line of the

& especially by those who are not conversant with the science in question. Thus the fitness of an axiom may partly depend on certain truths demonstrable independently of ~~those axioms~~ it. & then the uninstructed reader cannot be competent to judge of it. It may be said that in that case it ought not to be delivered till those things are demonstrated — true if only by a master. But ~~in boo~~ books are not complete masters & it behoves ~~the~~ beginners not to cavil at any thing which is not repugnant to their reason, till they see what use is to be made of it.

forces direction, & arise from tending from the centre to that point.

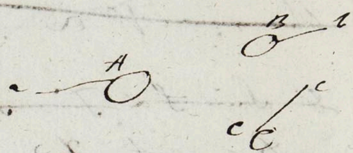
I shall confine the illustration of this axiom to the case of weight (~~ordinary~~ masses of matter acted on in it tiers) both in order to avoid all ~~ambiguity~~ disquisition here respecting the word force, and because in that case it is the only one we have experience in ~~within~~ all the force of the argument derives from experience is indirect in other cases, and appl. obtain upon us only in proportion to the habitual accuracy with which we understand the word force to mean something which produces some effect to that of gravity. The substitution of general forces in

× General acting in any direction is easily made afterwards

× In case of Equilibrium gravity may be made to supply the place of any supposed force, by means of strings & pulleys fit to something independent of all the machines whose properties are enquired into, for the action against that ~~string~~ part of the Machine to which one end ~~or support~~ of the string is supposed attached is the whole action of the wt at the other end & is independent of the direction in wt gravity acts on that wt. But in case of movement ~~the~~ such substitution by introducing new masses of matter which must necessarily move with the machine moves will necessarily produce a new modification of movement

2d Sheet

I say that if Bodies A, B, C, &c do ~~move~~ in set off from rest ~~by the~~ action of X mechanical causes, in direction Aa Bb Cc &c



with initial velocities proportional ~~to these~~ lines, they are in fact acted upon by the forces Aa Bb Cc, whatever be the metaphysical causes to which we attribute these forces, ~~& that~~ whatever be the connexion of the bodies & the machinery by which they move; for the force (the result of all that act on a body) is no way to be ascertained but by the effect it produces

X as distinguished from the action of animals or the like

X These forces may be the result of various causes & ~~imagined forces as cohesion propagation of motion, tension &c~~ on enquiring into which ~~we cannot at~~ has no concern here

~~&~~ consequently since by the doctrine of the forces springing all that at the same point ~~[illegible]~~ keeps a body at rest, these bodies would be kept at rest if each of the motions or force was opposed by an equal & equal motion or force in a contrary direction; if each of the bodies be acted on they must every one be opposed at the same time; because by this connexion ~~between the~~ bodies one body cannot be at rest without the rest

$\frac{s}{mt}$ $\frac{2m}{T}$

Suppose the velocity generated by gravity in a certain portion of time ~~(which call unity of time)~~ D be ~~called 1. Suppose the space passed in that time (from rest) m~~ is such that ~~then V ∝ s, $V = \frac{T}{D}$~~

~~& since s ∝ T² $s = \frac{mT^2}{D^2}$~~

would carry the body over ~~...~~ the space C in that time

~~then since $V \propto T$~~ ∴ V:

By the Vely of a body I shall in future mean the space its motion continued uniformly for a time D would carry it over.

Suppose the Velocity generated by gravity in that same time D to be C (i.e. suppose $\frac{1}{2}C$ the space fallen thro' from rest by gravity in the time D; ~~then Vely by gravity because~~

~~$V \propto T$ Vely by gy $= \frac{CT}{D}$, & because $s \propto TV$, $s = \frac{TV}{2D}$~~

& if the Velocity generated by any other force in same time D was $\dot{C}$ ~~then that other~~ force would be $\frac{\dot{C}}{C}$ × gravity which I call $\dot{g}$. ~~$\frac{\dot{g}C}{g}$~~

~~the Velocity generated by it in time T (which ∝ $\dot{g}T$) would be $\frac{\dot{g}CT}{g \times D}$~~

∴ since $V \propto \dot{g}T$ & $s \propto TV$

we have $V : C :: \dot{g}T : g \times D$ & $V = \frac{C\dot{g}T}{gD}$. also $s : \frac{1}{2}C :: TV : DC$ ∴ $s = \frac{TV}{2D}$

$s : \frac{1}{2}C :: TV : DC$ & $V : \dot{C} :: \cancel{TD} : TD$

$s = \frac{TV}{2D}$ $V = \frac{\dot{C}T}{D} = \frac{\dot{g}C}{g}\frac{T}{D}$

combination 3 out of ~~abcd~~ ~~abc~~ acd
from these two or between the 4 Quantities
F, S, T, V, we may obtain the 2 Eqs. between the
remaining two combinations of 3. viz the Eq. between
F, S & T, & that betw F, S & V. that is
S the 2d is

1st (M) if we make 1 one second & call it (1) $\frac{1}{2}$ ~~became~~
will be about $16\frac{1}{12}$ feet if we put this value of C = m
& Gravity call 1 the 4 Equations become

$$S = \frac{TV}{2} \; ; \; V = 2mFT$$

& under this form we shall

$$S = mFT^2 \; ; \; S = \frac{V^2}{4mF}$$

use them in future

By the ~~momentary~~ ratio of the two moving forces is meant the ratio of the
momentum generated in given time is the ratio of the two bodies
moved & that of their two velocities generated ~~due~~ ~~lessening~~ jointly
but this latter ratio is that of the accel. forces
∴ the Moving force (M) is as the body moved (A) & acc. force (F) jointly
& I suppose M = AF
The moving force is what is called the Wt of a
body.
& When = moving forces directly oppose each other they
destroy each others effect. Examples

But to explain what circumstances would authorize us to say that equal forces were applied to A B, &c

placed in the time = A

Suppose a body = A & instead of being acted upon by gravity acted upon by a force F in Direction = A

if the bodies having been previously accelerated

|| Or More generally suppose that the points A B C &c with certain velocities & be there accelerated by the forces

say F' F'' &c

be called 2 F F'' &c

& i.e. if $\frac{V}{t}$ for the diff^t bodies

~~[illegible] the acceleration be not uniform~~

addition to (M)

Cor. $v' = 2m F t'$ & $v = 2m F t$

Def^n of the force when the acceleration is not uniform

Let v be the increment of the Velocity (V) received during the time t' (u) & v'' the incr^t of Vel. by gravity (1) during the same time is $2mt'$. (cor)

& call A F the force affecting the time F

then F : gravity (1) :: Limit $\frac{v}{t}$: Limit of $\frac{2mt'}{t}$

:: Limit $\frac{v}{t}$: $2m$

:: $\frac{v}{t}$: $2m$

$\therefore F = \frac{\cancel{2m}\, v}{2mt}$

Cor. $v = 2m F t$

Op. tr.

When a body changes its state of motion or rest by the action of other bodies or by gravity it is said there force communicated to it & the force communicated is $\frac{v}{t}$

Suppose a system of ~~bodies~~ weights ~~applied~~ to ~~attached~~ any machine, & not in equilibrium suppose them kept at rest at first by any extraneous obstacle ~~force~~, as ~~the hand~~ ~~or~~ supported by the hand, or any other way upon removing the obstacle ~~each~~ the weights move in their different directions according to the nature of the machine & any of them of weights each with a certain initial velocity (vide page) i.e. so that ~~the~~ ratios of the initial contemporary spaces one to another in same passed over by each body ~~or~~ ~~another~~ have certain limits

i.e. so that the ratio of initial space passed only one body to the initial space passed over by any other of the bodies in the same time will have a certain limit, which limit called the ratio of the initial velocities of the bodies

This ~~lim~~ initial vel. is as limit of $\frac{\text{space passed in any time}}{\text{that time}}$

whereby limit is meant the value to which this fraction approaches, ever limits when the time is diminished ever limits

if $s\ s'\ s''$ ~~&c~~ be the space passed by the different bodies in time t

this initial velocity $\propto \frac{s}{t}$

this limit (limit of $\frac{\text{space}}{\text{time}}$) is called the force communicated to the body.

Suppose limit of vel in space A goes : space by gravy in same time :: F : 1. then force comd to A : force of gravity :: F : 1

& calling force of gravity 1, force comd to A = F

This ~~forces~~ comd to A results from the gravity of the body & its connexion with the others by means of the machine

Suppose the bodies A B C &c & these forces F F' F'' &c

then these bodies do really move ~~as if they were~~ exactly as they would if they were free & independent both of each other & of gravity & acted upon by the forces F F' F'' &c

Ex. A a C B b   B = 2A : = distances

B descends thro Bb & A ascends with equal velocities thro Aa

& whole Acc force comd to B = $\frac{B-B}{B+A}$ (gravity being 1)

& Acc force comd to A is the same but in contrary direction

Now if A & B were intirely independent & ~~the said force~~ ~~were one~~ instead of being acted upon by G. B was acted upon by the said force in Dn Bb

& A instead gravity by same force in direct Aa the same motions would take place

15

~~of since the Phenomena are the same in the two cases & that causes are measured by their effects, we may consider the causes as the same in both cases~~

1 If two forces oppose each other at the same point in opp. direction the effect is the ~~their difference~~ same with respect to any moment of that point or elsewhere they generate as if their difference acted alone in the direction of the greater.

Now if ~~there in~~ the bodies were so independent & acted upon by other forces Bb Aa, what forces applied to them would ~~to~~ destroy all motion or prevent ~~and only~~ the acting on the same point destroy each others effect is

forces bB(-p) to B, & aA(p) to A. therefore also in the other hypothesis, when the bodies are connected together & the motion caused by gravity, yet since ~~equal~~ phenomena are so far as regards momentum i.e. the real effects are the same as before & that the effects are produced by the causes, & i.e. destroyed by the causes, in this other hypothesis being, the same forces

bB to B & aA to A will destroy prevent the motions & produce equilibrium

It may be objected perhaps that the phenomena are not the same in the two cases. for that in one case there is a pressure at C. & none in the other.

800

I answer that by the doctrine of the composition of forces this pressure is in effect a nonentity for it produces no motion in C. &c. i.e. is counted as opposed by a contrary & equal pressure & contrary & equal pressures at the same point produce the same effect as no pressure at all.

I shall shew in the sequel that the phenomena of gravity may be considered as an ultimate case of impact. & than by means of the ~~all the~~ common law of impact. that no momentum is lost in any direction ~~is~~ by the mutual actions of bodies ~~…~~. ~~…~~ it will be very easy ~~…~~ all contiguity from distance pressure

Another objection may be ~~…~~ that the two hypotheses can not be considered as effectually the same because in the one (viz. where the bodies are independant) a force as B C would produce motion in B, carrying it & move on. whilst it would not in the other hypothesis.

Since it is necessary in the other hypothesis to oppose the motion of all the bodies otherwise they act one on the other. their connexion hinders from keeping one body at rest ~~…~~ whilst the others are left unopposed. but oppose all the motions of one simultaneously & the difficulty vanishes & the objection with it

1 Defn. If a body moves uniformly its velocity = the space it passes thro in any time divided by that time.

2^d^ Def. If a body moves any how either uniformly or variably, the velocity at any point = the limit of the space passed over reckoned from that point ~~divided~~ by ~~the~~ time where by limit is meant the value to which this fraction approaches ~~indefinitely~~ when the said space is diminished sine limite.

Thus if a point describes the line A — m P n — B

AB, the velocity at P = the limit to wh of $\frac{Pn}{\text{time thro } Pn}$ approaches as n moves up to P ✕ is taken nearer & nearer to P sine limite.

It is also equal to the limit to which $\frac{Pm}{\text{time thro } Pm}$ approaches when m is taken nearer & n s.l. to P.

[✕ in other words ~~that~~ it equals the limit to which $\frac{Pn}{\text{time thro } Pn}$ approaches when n moves up to P.

3. If these two limits be different two velocities belong to the point P.

4. If the value of the said fraction diminishes sine limite there is no velocity at P.✕

[a body may ~~[illegible]~~ have a velocity from one ~~limit~~ of the limits & none from the other. or none from both. Observe the body is not supposed to stop.

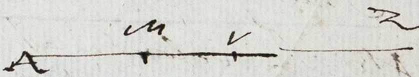
[V. in any direction defn.]

Observation. In a uniform motion (in uniform Space & Time) the rate or velocity is had by comparing the space with the time & one space gives the same result as another. Thus if a body moves uniformly [at the rate of 10 feet per second], & we take 30 feet & we find it passed over in 3 seconds then ~~&~~ the velocity indicated is 30 feet per second; if we take 20 feet we find it passed over in 2 seconds & the velocity indicated is 20 feet ~~per second~~ in two do. which is the same as 30 in 3 seconds.

But if the ~~m~~ motion be variable ~~the~~ one velocity will be indicated ~~will be diff~~ for by one space & time & a diff. another by another & that even if we take the diff. spaces to begin at the same point.

For ex. ~~Let S vary & let~~

Let the space described by a point moving along the line AZ ~~&~~ & ~~vary~~ vary as the square of the time it employs. & let AM be described in a time D.



What is the velocity indicated at M.

Take MV such that time thro' MV = D; then MV = 3AM (by hyp) $\frac{9}{4}$AM

~~Take MV = 3 AM time thro MV = D~~

& vel. indicated is 3AM in time D.

I take (the time = $\frac{1}{2}$D; then) MV = $\frac{5}{4}$ of AM. ~~Time be $\frac{1}{2}$D~~ $\frac{1}{8}$ $\frac{16}{9}$

& vel. indicated is $\frac{5}{4}$AM in time $\frac{1}{2}$D i.e. $\frac{5}{2}$AM in D (less than before)

I take (time $\frac{1}{3}$D) MV $\frac{7}{9}$AM ~~Time be $\frac{1}{3}$D~~

& V indicated is $\frac{7}{9}$AM in $\frac{1}{3}$D or $\frac{7}{3}$AM in D less than in the case

P 3

(invent; assume time)

k^{ths} take MV $\frac{2n+1}{n^2}$ AM ; time is $\frac{1}{n}$ D

Velocity indicated is $\frac{2n+1}{n}$ AM in D or $2AM + \frac{1}{n}AM$ in T.D. & the greater n is, the less does this velocity differ from 2AM in Time D.

The indicated velocity then is a variable quantity in this case, but perpetually decreases & approaches to a certain limit viz 2AM in D & this limit is called the velocity at M.*

We may define then the velocity to be the limit of the indicated velocity.

The velocity indicated by the space previous to M will ~~be~~ have the same limit (which will be $\frac{2n-1}{n}$ AM in D). & This identity of the two limits ~~always~~ will always happen when the time is an algebraic function of the space.

* when S ∝ T² The motion accelerates. Therefore it is natural (according to the popular undefined notion of velocity) to say that the V indicated by the space succeeding a point is always gr than the real ~~at~~ velocity and that the velocity indicated by the preceding space at any point is always less than the real velocity ~~at any preceding point~~ but that there is a certain velocity between the two which is the real ~~&~~ & ~~thus~~ the V is determined by the above test. For the vel is to be between $2AM + \frac{1}{n}AM$ ind & $2AM - \frac{1}{n}AM$ ind whatever n be & no other magnitude whatsoever gr or less than 2AM can universally be between ~~them~~ for all possible values of n

* ~~Take space $\frac{2n-1}{n^2}$ AM time is $\frac{1}{n}$ D velocity indicated is $2AM - \frac{1}{n}AM$ in D~~

~~so that there is a true velocity~~

4

The popular idea of Velocity is more simple than the the mathematical but fails of being applicable always. Whenever it is applicable, the Mat includes it

& The same may be said of the popular idea of ratio & equality of ratio ~~is much~~ & This simplicity of idea is so fascinating ~~that to~~ both in this & other cases that tis difficult (nay, it seems, impossible) to convince the world (the mat. world) of the necessity of mat defns of their abstract terms. By virtue of something in the word itself they proceed to demonstrate something concerning that word & then something concerning the many instances where that word is concerned.

All the & English followers of Newton have done this with the word velocity. Newton himself was entangled in this snare & gives a most striking proof of it when answering an objection to his doctrine of limits. Vide Principia Scholium at the end of the 1st section. His words are "

Would not any one suppose from these words that no law of motion could be laid down but that the body should have a certain velocity at each point? Whereas in fact tis very easy to ~~make~~ contrive an hypothesis ~~for~~ for a body ~~to~~ moving over a line, ~~such~~ an arrangement of the time & space so that it shall be impossible to say (either by virtue of the supposed meaning of the word velocity or by the mathematical defn $v = \frac{s}{t}$) what the velocity is at any point & even whether the motion be accelerated or retarded ~~in the~~ the passage during any proposed part of the line

Vide o[illegible]

5

^ It is foreign to my purpose to enlarge upon this subject at present. either by pointing out what parts of Mathematics have their demonstrations conducted along these slippery Metaphysical roads or by pointing out the true roads but I think that a severe reform accompanied with a full explanation of its use & of the insufficiency of popular notions is of all things the most to be desired at this time for the mathematics.

here

4. The momentum of a body ~~is~~ is its quantity of matter ~~[illegible]~~ ~~Vol. V~~ × its velocity (V) ~~Momentum [illegible]~~

~~Any~~ proportion

5 The limit of $\frac{\text{Vel generated } \sout{\text{in a body}}}{\text{the time in which tis generated}}$ × is called the actual accelerating force on that body at the beginning of that time.

× Any limit of a fraction is always meant the value towards which it approaches when the numerator & denominator are diminished ~~ad~~ in limits ~~one~~ extremity of the quantities concerned is supposed called the actual first, & the other brought nearer & nearer to it giving limits to that limit in question belongst the first point. In general, unless the contrary be specified, that extremity which precedes in order of time or space is supposed the first extremity & [illegible] in the succeeding increments are taken.

6 the Limit of $\frac{\text{V of destroyed in a body}}{\text{time in wh tis destroyed}}$ is called the actual retarding force at the beginning of that time

For the sake of Generality expression a vel. destroyed is called a vel. generated in one direction is called a vel. destroyed in the contrary & a retarding force in one direction is called an accelerating ~~force~~ in the contrary

So ~~[illegible]~~ likewise a velocity in one direction is called a vel. in the contrary direction. & This generalization method is employed ~~[illegible]~~ ~~[illegible]~~ for all names & for all cases where it can serve to simplify diminish the number of algebraical formulas

Insert formulas && but first $T = \frac{v}{t}$

7. ~~The gravity of mass × accelerating force~~ actual

~~the~~ ~~that~~ that acc force which would be (by hyp) be the actual acc force if ~~acting~~ on the body under certain circumstances is called the hypothetical acc force. If it be the acc accelerating force that would take place if the body were left free & independent of the other bodies or obstacles, this also called the free force

Ex Let A & B be two weights of which B is the heavier attached at each end of the lever revolving about its middle point which is supported fixed ~~if the weight be~~ if the lever be placed horizontally & left to the action of gravity, B will descend & A will ascend. The actual accelerating force on B acting or the actual accelerating force indicated by the movement which really obtains will be $\frac{B-A}{B+A}$ of the actual accelerating force of a body freely falling by gravity & ~~in the~~ it will be in the direction of gravity.

the a acc force on A is the same but in contrary direction

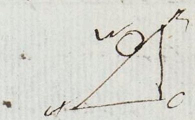
The hyp or free force on each is gravity

Ex 2d let the fixed point be C between A & B

then ac. acc. force on B = $\overline{-A\cdot AC + B\cdot BC} \times \frac{BC}{B\cdot BC^2 + A\cdot AC^2}$ & is in direction of gravity; that on A is to this in the ratio of AC to BC & in a contrary direction

the free force are those of Gravity.

Ex 3. Let a weight W be left to slide down the inclined plane BA.



the accel. force is $\frac{BC}{AB}$ × Gravity in its direction BA. the force is ... of gravity.

Ex 4. Let W be attracted ~~...~~ an equal weight hung freely by means of a pulley at B so placed that the string BW may be ∥ to the plane. the accel. force on W is $\frac{AB-BC}{2AB}$ × Gravity in its direction WB.

if ~~the circumstances~~ were changed & instead of being attached to another weight W were left free to slide down the plane its ~~...~~ with respect to the system in question the force would be $\frac{1}{2}$ Gravity & this is a hypothetical force if it was left entirely free from obstacles it would fall by gravity which is ∴ another hypothetical force ... its free force.

Ex 5. C A B D

Let there be three weights on the same side of the Center of motion. The accelerating force on B is

$$\frac{A\cdot CA + B\cdot CB + D\cdot CD \times CB}{A\cdot CA^2 + B\cdot CB^2 + D\cdot CD^2} \times \text{Gravity}$$

if D be supposed left out of the system it would become A . CA which ∴ is an hyp. force. if A be also left out it becomes B's by Gravity another hyp force

8 Defn When the circumstances which determine an hyp force ~~are~~ to any body are such as not to involve or require the motion of any ~~other body~~ mass of matter such hyp. force is called an independent acc. force

It is manifest that ~~x~~ every free force is an independent force

Thus the 1st hyp force in Ex. 1 is an independent force. So is the last in the last example.

9. The quantity of matter in a body x the acc force is called the acc moving force.

10 The quantity of matter ~~x~~ the hyp a f is called the hyp. acc force.

10½ Vide page 9
11 The quantity of matter x force a c is called the force moving force

~~The~~ q. of matter into independent ~~moving~~ acc force is called the independent moving force.

11. Defn. All the preceeding forces intended by the above definitions were of appt. to certain vel. generated during the times which are punctual, & are assumed equal to the said fractions divided by the velocity generated in time 1 by an ac force & which a c force may be called the standard. Thus all ac force will be numbers & every such force will be that number which would be mechanics in the same ratio that force & standard force in that number which ratio to unity = ultimate limit of Vel. gent in some f the ac acc f time in wh. the gent

the Vel. gent by standard force
time in which the gent

Vide P. 10

Vid page

10 ½ If the acc.l accel force be as constant ~~~~ 9

is if ~~any time~~ the passage over any space i.e. if $\frac{v \text{ generat}}{\text{tim}}$ have always the

same ~~~~ for any during the description of that space

its actual value is the same as its limiting value is constant &

& conversely in such case the acc.l force which is proportional to

~~~~ $\frac{v \text{ generated}}{}$ is proportional to its equal $\frac{v \; g^n}{\text{tim}}$

~~& conversely~~ ~~~~ that time

X proof ~~of the converse~~ ~~hyp~~ $v \propto \frac{v}{t}$ if constant $= a$

$\therefore v = at \quad \therefore v = at$ ~~+ c~~ which wants no corr. because

v & t are supposed to begin together $\therefore \frac{v}{t} = a \ldots$ but any increment of v & t of it

$v + v' = at + at' \quad \therefore v' = at' \quad \therefore \frac{v'}{t'} = a = \frac{v}{t}$

but the force may be in ? by the velocity that ~~~~ forward $\frac{\cdot}{\cdot}$ the time so yet not constant

Conversely if $\frac{v}{t} \propto \frac{v'}{t'}$ each is constant ~~~~

but whether ~~~~ may vary as etc as for $\frac{v'}{t'}$ & yet the acc. for be variable

~~$\frac{v}{t} = x \quad v = tx \quad v = tx + it \quad \frac{v}{t} = x + \frac{it}{x}$~~

$\frac{v}{t} \propto \frac{v}{t} = \frac{at}{v} \therefore$

~~for~~ $\frac{v}{v} \propto \frac{at}{t} \therefore Lv = a \, Log \, t + ~~Log~~ \ldots = a \, Log \, t + Ld$

$= a \times Ldt$

$\therefore \frac{it}{x} = dx$

$\therefore v = ~~~~ \propto t^a$ ~~~~

& the force is constant if $a = 1$, otherwise

$\therefore t = \frac{x}{i}$

~~~~ $\frac{a}{} \times at^{a-1} + \ldots$

$\therefore \frac{v}{t} = \frac{x^2}{i}$

~~~~

If we assume $\frac{v}{t}$ to vary as $\frac{v}{t}$ also we assume ~~constant~~ for some $v = dt^a$

$v = dat^{a-1} \ldots \& \frac{v}{t} = d$
~~~~

Vide end of page 8

or the ratio $\frac{L \cdot V \cdot G \text{ in conseq. of the acc. force}}{\text{time}} : \frac{VG \text{ by standard f}}{\text{time}}$

Let m = space from rest by standard force (1) in time 1. then $2m = V \cdot S^{tard}$ ~~& V gen in time 1 / time 1 = 2m & this doubly~~

~~for VG 2m becomes 4~~

Then (if force be uniform) $S = \frac{TV}{2}$ (1) $V = 2mFT$ (2)

hence by elim. V we obtain the equation between S F & T

viz. the eqn $S = mFT^2$ (3)

& by Eq. 3 we have $V = \frac{4mFS}{V}$ or $V^2 = 4mFS$ (4)

~~if F be not uniform~~

whether F be uniform or not we have $\dot{S} = V\dot{t}$ & (art)

$\dot{V} = 2mF\dot{t}$ (def) hence $\dot{V} = \frac{2mF\dot{S}}{V}$ ~~or~~ $V\dot{V} = 2mF\dot{S}$

also $\dot{V} = \frac{\ddot{S}}{\dot{t}^2}$ (if t be constant) & $F \left(= \frac{\dot{V}}{\dot{t}}\right) = \frac{\ddot{S}}{\dot{t}^2}$.

use A instead of L

In all these expressions F is used for $\frac{\dot{V}}{\dot{t}}$

i.e. if we employ $2mF$ for $\frac{\dot{V}}{\dot{t}}$ in q. w. define the force (which $\propto \frac{\dot{V}}{\dot{t}}$ to be = to $\frac{\dot{V}}{\dot{t}}$ ÷ 2 the space by standard force in 1")

3 29

If a single unconnected body be ~~fixed to~~ acted upon by a force ~~from~~ some ~~due~~ tending to a point, & this body ~~to~~ ~~for~~ moves along a tube (void of friction & of continued curvature or strait) while another body moves strait & unconfined to or from the center of force, & the velocities be equal ~~to~~ ~~at~~ in one case of equal distances from the center of force, they will always be equal.

If a number of bodies so move, attracted to different centers of force, some setting off from rest

---

~~but if~~

Now when c is positive i.e. when direction of the body's motion is favorable to force, $\int PAci$ ~~$= \frac{pv^2}{2}$ where v = the velocity~~ of each body moved along a tube S independently of the other bodies, without the $PAci = PVv$

& $\int \quad = \frac{pv^2}{2}$

& when c is neg we should have $\int = -\frac{pv^2}{2}$

$\therefore$ adding we have $\int + \int + \int + \&c = \frac{pv^2}{2} + \&c - \frac{pv^2}{2} + \&c$

but $\int\int\int = \int$ above $\therefore$ &c

If the forces tended to centers ~~there~~ first & were to equal at = distances

same velocity would be acquired in moving from rest from one given

distance ~~[illegible]~~ to another whatever was the path; all being equiv. to that acquired

in only the same approach to the center in a strait line towards it

In such case ∴ if the attr. of the body to cr be called

m in x & the recess of those that recede n n' &c

$\int - \int - \int$ &c $= \frac{D}{2} \times V^2$ thro $m + \&c - \frac{C}{2}. V^2$ thro $n - \&c$

if each force be constant V^2 thro $m = dAm$; &c

in this case $\int\int\int = P\,dAm + dA'm' + \&c - dAn - dA'n' - \&c$

if all these forces be = $\int\int\int = \frac{dA}{2} \times Pm + P'm' \&c - Cn - C'n' - \&c$

if all be = const. & $\int\int$ ~~$\int\int\int = dA$~~ $Pm + \&c + Cn - \&c = \overline{P + Q + R + \&c} \times$

~~[illegible]~~ ac ~~[illegible]~~ of cr of gravity from from . . . †

∴ in this case $\int\int\int = \frac{dA}{2} \times \overline{P + Q + R + \&c} \times$ accept of C of G

† $2g \times Sum \times h$

× $Pm + \&c - Cn - \&c$ is repr by $S.\frac{\overline{PAV}}{2}$

if PQRV = the limit of the area from V upwards ad infinitum, i.e. if $c^2 = \frac{4sr}{n}$, the vel. is said to be that acquired by falling from an infinite distance

if PQRV be g^r than that limit, or c^2 g^r than $\frac{4sr}{n}$ the vel. is said to be that acquired by the bodys being projected from an infinite dist with ~~a~~ some vel^y $\sqrt{c^2 + \frac{4sr}{n}}$

To find the point A whence the body fell, when the vel. is less than that acquired from an inf. distance. i.e when $\frac{r^{n-1}}{4s}c^2 + \frac{1}{n} \cdot r^n$ is positive

When $x = CA$, area $= 0$

$$\therefore \text{ put } \frac{Dr^{n-1}c^2}{4s} + \frac{D}{n} \times r^{n} - x^{n} = 0$$

$$\& \frac{D}{n}x^n = \frac{Dr^{n-1}c^2}{4s} + \frac{D}{n}r^{n} = \frac{D}{n}p^n$$

$$\& x^n = \frac{nr^{n-1}c^2}{4s} + r^{n} = \frac{nc^2r^{n-1} + 4sr^{n}}{4s} = p^n_{CA}$$

$$\& x = \sqrt[n]{\frac{nc^2r^{n-1} + 4sr^{n}}{4s}} = p = CA$$

If $n = 0$ put $\frac{Dr^{n-1}c^2}{4s} + D.L\frac{x}{r} = 0$

then $L.\frac{x}{r} = -\frac{r^{n-1}c^2}{4s} = \frac{c^2}{4rs}$

$$\& L.x = L.r + \frac{c^2}{4rs} = (\text{suppose})\ L.p$$

$$\& x = p = p = CA.$$