On parallel Lines. TH19817 1

That the diligent researches, which have been made after some unexceptionable mode of demonstrating the properties of parallel lines, have been hitherto ineffectual is well known to the Mathematical world. The acknowledged Genius & Skill of many, who have been employed in this pursuit, when contrasted with the apparent simplicity of the subject has given rise to the opinion that further research is in vain — & has occasioned an excellent Mathematician* to admonish the curious not to waste their time on what has baffled the ingenuity of the most eminent Geometricians.

But it should be recollected that researches of this kind even where they produce nothing satisfactory with respect to their ultimate object, are yet by no means to be considered as fruitless, nor the time employed upon them as totally misspent. For by minutely laying open to his mind the various sources of error; — by scrupulously examining the logic of each argument, by rigidly rejecting every the most trifling step which is not dependent on principles already demonstrated true — any thing that he does not choose to regard as an axiom, the persevering Enquirer will gain both a greater facility in the detection of error, & a greater promptitude in the discovery of truth.

Led to the enquiry at first from curiosity, & in-

* Professor Playfair

duced to continue it, even after many failures, partly from the experience of its incidental utility, partly from the uneasiness which is felt at relinquishing any pursuit, which may yet be successful, I long since discovered a method, of establishing the doctrine of parallel lines upon any one of the four following axioms

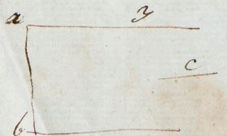
1st. "If the indefinite straight lines ay bz be perpendicular to ab, & the line cd lying between them be parallel to to a y, it will, when produced, meet either a b, or b z."

2d "If ay, bz be perpendicular to ab, & a line be drawn from any point in a y at right angles to it, it will meet bz."

3d "If ac, cd, de, &c be all equal, & each be inclined to the next at equal angles y c d, [illegible] & the same way, the points a, c, d, e, &c ad infinitum cannot all be equally distant from the straight line b z."

4th. If ay, bz be perpendicular to ab, & the line cd lying between them be ~~perpendicular~~ parallel to bz, & ce df be drawn perpendicular to ay, cg, dh perpendicular to bz, ce, df ~~will be~~ proceed from cd on one side way, & cg, dh on the other side way"

fig 1st



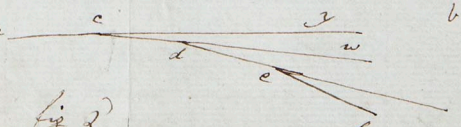
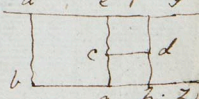


fig 2

fig 3



And I did for some time consider this as an improvement on the doctrine of parallel lines: but upon maturer consideration I was dissatisfied, being convinced that to admit any one of the above propositions as an axiom would be detrimental to the strictness* of geometrical reasoning: Without entering fully examining fully

It is not my intention to enter here into a disquisition on the nature of mathematical axioms, [although it presents a field, whose further cultivation might be far from unproductive; I shall only] observe that axioms ought to be founded on experience — not that I mean to say that Mathematical truths are to be inferred from any number of experiments in a similar manner as the assumptions in other branches of science — but axioms ought not to assume cases, the existence of which is not ascertained by our common sensations. — If we had not the evidence of our senses to assure us that there may be lines which cannot coincide in two points

* We are deceived into the conception of the self-evidence of the above propositions by the view (real or mental) of the correct figure; the hypothesis of the axioms leads us to conceive that any two will always preserve nearly the same distance from each other; & if they be drawn conformably to that conception it will be impossible to contrive to exhibit a figure at variance with the axiom but a figure may be so drawn as to countenance the supposition of the above axioms being false, & at the same time contain nothing absurd or incredible; thus for example we may suppose ey bz so to recede from each other, as (after a certain distance) to have the parts ef, gh justly represented in fig 4; where the four perpendiculars ae, bf, cg, dh are all on the same side of cd

fig 4

without coinciding altogether", the axiom "Strait lines enclose no space" would be a vain assumption. This axiom is, as it were, to right lines what definition the 5th B. 5th Simpson's or Playfair's Euclid is to equal ratios — the criterion by which we know what is meant by right lines; & what is usually called the definition of right lines "A right line is that which lies evenly between its extreme points" is only a loose phrase, whose use, if any it to excite in the mind a general & popular notion of what sort of lines is coming under contemplation — it is of the same nature as def" 3d Book 5th — The demonstrations which are founded on the above axiom "Strait lines enclose no space" stand or fall according as it is allowed or denied that lines of such a description may exist — That being granted, the conclusions are established universally for all cases where the lines are of that nature, & no more than this the demonstrations do not pretend.

If in like manner experience assured us that two strait lines could not be parallel, when a third line falling upon them made the sum of the two interior angles on the same side either greater or less than two right angles, the demonstrations of the properties of parallel lines as delivered in Euclid's Element would be legitimate ~~demonstration would require this addition~~ only we very often find that two strait lines are exactly equidistant in all their parts. But these are truths neither of that kind which is demonstrated by our immediate sensation, nor of that kind which can be universally demonstrated by experiments. —"

If is very well known that that the properties of parallel lines may easily be proved, if we allow that the line which continually preserves the same distance from a given strait line, must itself be [illegible] but this would be [illegible]