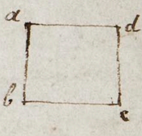
adopted or the following "~~Three points~~ Any number of 3 points may be found lying in one strait line, each of which shall be equally distant from a given strait line". If the reader should think that ~~this~~ the above axiom may more easily be brought to the test of experiment than the one I have adopted he will find it equally convenient to argue from ~~as~~ ~~that which I have adopted~~ — & ~~the~~ ~~that~~ may employ ~~&~~ nearly the same method. In order to expedite the demonstration of one proposition I have subjoined another axiom, which I have long thought worthy of a place in mathematics, an opinion which I am happy to find confirmed by so eminent & judicious a master of the science as Professor Playfair.

Axiom 1

It ~~is found~~ that in some cases that the angle b a d formed by the lines b a, a d being the line joining the extremities a & d of the two equal perpendiculars upon b c & b a being one of those perpendiculars, is a right angle



Axiom 2

When any figure is given, another may be supposed to exist exactly like it.

I have ~~perceived~~ long thought that this axiom might be employed very advantageously in the 1st part of Euclid. I have annexed it here in order to expedite ~~the~~ the dem of one of the following propositions

Prop. 1st

$$\frac{2ax\dot{x} - x^2\dot{x}}{\sqrt{2ax-xx}} = \frac{2ax^2\dot{x} - x^3\dot{x}}{\sqrt{2ax^3-x^4}} = \frac{\frac{3ax^2\dot{x}}{2} - x^3\dot{x}}{\sqrt{2ax^3-x^4}} + \frac{\frac{1}{2}ax^2\dot{x}}{\sqrt{2ax^3-x^4}}$$

$$\therefore fl = \frac{1}{2}\sqrt{2ax^3-x^4} + fl\ \frac{\frac{1}{2}ax\dot{x}}{\sqrt{2ax-xx}}$$

$n+1 = 2n+2$

$n = -1$

$x^n\dot{x}\sqrt{2ax^{2n+1}-x^{2n+2}}$ $\qquad x^{-1}\dot{x}\sqrt{2ax^{-1}-1}$

x^{-1}

$$fl.\ \overline{m.\,2ax-x^2}\big|^{\frac{3}{2}} = \frac{3}{2}m.\,2a\dot{x}-2x\dot{x}\sqrt{2ax-x^2}$$

$$-\overline{a\dot{x}-x\dot{x}}\times\sqrt{2ax-x^2} + a\dot{x}\times\sqrt{2ax-x^2}$$

$x^{1+n}\dot{x}\sqrt{2ax^{1+2n}+x^{2+2n}}$ $\qquad \frac{x\dot{x}-a\dot{x}}{\sqrt{2ax-xx}} + \frac{a\dot{x}}{\sqrt{2ax-xx}}$

$a : a+x = r$

$\frac{1}{2}a : \frac{1}{2}a+\frac{1}{2}x = r$

$\frac{1}{2}a : \frac{1}{2}a+x = 2r$

$$\boxed{-\sqrt{2ax-xx} + \text{cr arc}}$$

$$\frac{1}{2}\sqrt{2ax^3-x^4} + \frac{1}{2}a\times\overline{\text{cr arc}-\sqrt{2ax-xx}}$$

$\frac{z\dot{z}}{a+bz^2}$ $\qquad \frac{z^2\dot{z}}{a+bz^{-1}}$ $\qquad \frac{z\dot{z}}{az^3+bz^2}$

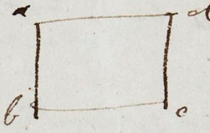
$\frac{\dot{z}}{z^2\times a+\frac{b}{z}}$ $\quad = \frac{z\dot{z}}{z^2\times az+b}$

$A^2 = B^2+C^2+2D$

$2D = B^2+C^2-A^2$

authority of so eminent a Math as Dr Playfair

Axiom 1



"It is possible ~~for the 4 angles~~ for b.a some times cd to be equal to each other & ⊥r to some base b.a & at the same time ~~for~~ that ∠ a be a right ∠."

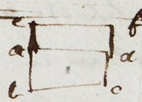
Ax. 2

~~Whatever~~ On any figure is ~~supposed to exist~~ given, another may be supposed to exist exactly like it.

Prop 1st &c

See page 7

Prop 8



Same proportion if ba be produced to e & cd to f so that ae = df, & e, f be joined the ∠s at e & f are right

9

If on PQ two equal lines QN, PM be erected ⊥r to PQ & M, N be joined the ∠s M & N are right angles.

Converse

6

10

If yz be every where at the same distance from vw yz is a strait line, & a line ⊥ to one of the two yz or vw will be ⊥ to the other.

11

If two strait lines do not keep the same distance they will meet.

12

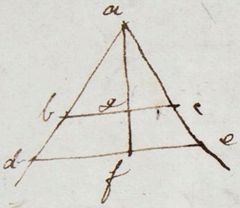
Cor. if two strait lines intersect, one of them will meet any third strait line [illegible] || to [illegible]

Hence follow all the properties of || lines

If in the same manner experience showed us that two lines could not be || ~~[illegible]~~ then a third line falls [illegible] make [illegible] two angles on the same side greater or less than 2 right ∠s, then the demonstration of the doctrine of || lines as given in Euclid would be legitimate but this is a truth not of that kind which is ~~known~~ [illegible] by an immediate sensation, one of that kind which can be ~~inferred~~ universally demonstrated by experiment.

It is very well known that the properties of || lines ~~cannot~~ any can only be demonstrated if we allow that ~~a strait~~ the line ~~passing through any~~ ~~point~~ which universally keeps the same distance from a given strait line will itself be a strait line but to concede this alone would be there would be too great a concession too much. Experiment ~~can only~~ may go so far as to shews that in some one or more cases ~~the distance of~~ two three or any other finite no of points being taken equally distant from a given strait line the line which ~~joins~~ ~~[illegible]~~ the points [illegible] will form one strait line. [illegible] if we can demonstrate that admitting this [illegible]
true in one instance it will necessarily be true universally [illegible] [illegible] to establish [illegible]

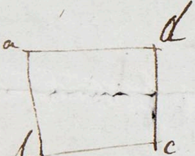
Cor 7



If abc ade be Isosceles Δs having a common ∠ at ~~the~~ vertex a, de is gr than bc

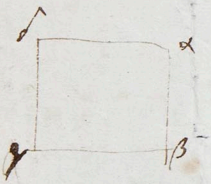
For if poss let de = bc bisect angle at A by af then ∠s at g & f are rt angles. ∴ if df were equal to bc df wd be equal to bg ∠s at d & b wd be equal by 1st prop which they are not &c.

1



If ∠s at b & c be right ∠ at side

∠ at a = ∠ at d

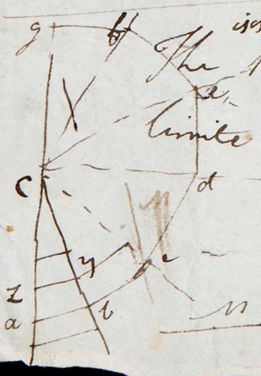


For let αβγδ be similar ~~...~~

γ upon b, β upon c, βα upon cd &c

1½ The converse

3



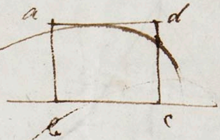
The isosceles Base zy of any angle zCy may be increased sine limite by increasing the sides Cz Cy.

For if poss let M be gr than any zy

~~take~~ Let zCy be either equal ~~...~~ n^{th} of 2 right angles or less than n^{th} but ~~...~~ than $n-1^{th}$ take yCw = zCy; &c

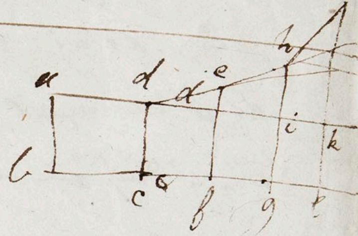
Again

The two equal lines ab dc Lr to bc being joined by ad the angles at a & d are in some one or more instances found to be right angles



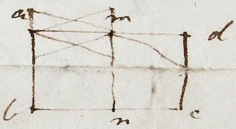
4

If L b & L c be right & ab = dc
L a is not gr than right L



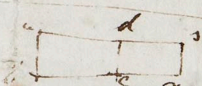
5

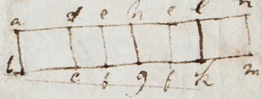
If ab = dc & the 4 angles of the figure abcd are rt & from [illegible] be drawn Lr to bc mn = ab & that more right



6

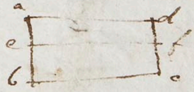
Iisdem positis if bc be produced beyond to r & rs be drawn Lr to br meeting ad produced in s rs = ab [illegible]





7

Iisdem positis if ae be taken equal df & the points e f joined L at e & f are right



for prop 8 see page 5

equal to [illegible]
will form [illegible] [illegible] [illegible] [illegible] [illegible]
[illegible] in one instance it will [illegible] [illegible] [illegible]

I was dissatisfied, being convinced that to admit any one of the above
propositions as an axiom would be detrimental to the strict ~~na~~ ness
we are deceived in the ~~belief~~ conception of the
~~ture~~ of geometrical reasoning. —— It is not my intention
self evidence of the above propositions by the view (real or mental) of the correct
to enter here into a disquisition on the nature of mathematical
figures & the ~~[illegible]~~ by position of the axioms leads us to conceive that
axioms, altho it presents a field whose further cultivation would
ay & bz will always preserve nearly the same distance from each other, & if the figure they be
be far from unproductive — I shall only observe that axioms
drawn, conformably to that conception it will be impossibly to contrive ~~a~~ to
ought to be founded on experience — not that I mean to say that
exhibit a figure drawn in conformity with the axiom (as being false) but a figure may be ~~[illegible]~~ so drawn as to
mathematical ~~axioms~~ truths are to be inferred from any num-
countenance the supposition of the above axiom being false, let the same figure contain
ber of experiments like the assumptions in other sciences — but
nothing absurd or incredible; thus for example we may suppose ay & bz to
axioms should not assume cases the existence of which is not ascertained
recede from each other, ~~but~~ as to have (after a certain distance) the parts
by our common sensations. If we had not the evidence of our
ef, gh justly represented in (fig) where the [illegible] perpendiculars ... are all within
senses to assure us that there may be lines, two points in one of
some side of cd;
which coming in contact with two points in another, the intermediate

points of each are necessarily in contact, the axiom "Strait lines

enclose no space" would be a rash assumption. This axiom is

in some measure
as it were to right lines what Defn 5th Book 5th Playfair's Simpson's Euclid

is to equal ratios, — a criterion by which we know what is

meant by right lines; & what is usually called the definition of

right lines is only a loose phrase whose use, if any, is to excite

in the mind a general & popular notion of what sort of lines is

coming under contemplation — tis of the same nature as defn

3d Book 5th. The demonstrations which are founded on this axiom

"Strait lines enclose no space" stand or fall according as tis, allowed ~~grant~~
that being granted,
~~ed~~ or denyed that lines of that description may exist. — the conclusions

are established universally for all cases where the lines are of that nature, & to more than this the demonstrations do not pretend.

If in like manner experience assured us that ~~two~~ two strait lines could not be parallel when a third line falling upon them made the sum of the two interior angles on the same side either greater or less than two right angles, the demonstrations of the properties of parallel lines as delivered in Euclid's Ell. would be legitimate. But this is a truth neither of that kind which is demonstrated by our immediate sensations, nor of that kind which can be universally demonstrated by experiments.

It is very well known that the properties of parallel lines may easily be proved if we allow that the line which continually universally preserves the same distance from a given strait line must itself be ~~a~~ strait ~~line~~ — but this would be

being by no means self evident × in its course from a to d

too great a concession — Experiment may go so far as to

taking

the a point equally distant from the strait line for ought we can tell, ~~...~~ of the points a d

shew us that that in some one or more cases the line which joineth ~~...~~ ~~...~~ equally dis- ~~two~~ three, 4 or

tant from bc determined & consequently the line a c d ~~...~~ any pointing which is ~~...~~ equally distant

any other ~~but~~ number of points being taken equally distant

from bc with a or d ~~...~~ may be a curved line

from a given strait line, the lines which connect the said

but it can assist us no

points will form one strait line — ~~...~~ further. If there-

but if experiment shall shew that this or some similar circumstance does take place in

upon supposition of its being

fore we can demonstrate that ~~allowing this to be~~ true in one

instance & by geometrical reasoning we can dem of the above circumstance obtaining being shown in

instance, it must necessarily be true universally, then we shall be able to establish the doctrine of parallel lines. — &

similar to

this is the method I have pursued in the subsequent pages. I was in some doubt whether to employ the axiom I have there